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$$-2 = \frac{c_1}{a_1 \rho \cos \theta + b_1 \rho \sin \theta} + \frac{c_2}{a_2 \rho \cos \theta + b_2 \rho \sin \theta}.$$

Transforming back to rectangular co-ordinates, we have

$$2(a_1x + b_1y)(a_2x + b_2y) + c_1(a_2x + b_2y) + c_2(a_1x + b_1y) = 0$$

for the locus of P . This equation represents a hyperbola passing through the vertex A . Hence the intersection of this hyperbola with the base CB will give R , and PR produced will give T .

CALCULUS.

248. Proposed by REV. R. D. CARMICHAEL, Anniston, Ala.

Evaluate $\int_0^{\frac{1}{2}\pi} \sin nx \cot x \, dx$, where n is a positive integer.

II. Solution by FRANCIS RUST, C. E., Pittsburg, Pa.

$$\sin nx = n \cos^{n-1} x \sin x - \left(\frac{n}{3}\right) \cos^{n-3} x \sin^3 x + \left(\frac{n}{5}\right) \cos^{n-5} x \sin^5 x - \dots$$

$$\therefore \int_0^{\frac{1}{2}\pi} \sin nx \cot x \, dx = n \int_0^{\frac{1}{2}\pi} \cos^n x \, dx - \left(\frac{n}{3}\right) \int_0^{\frac{1}{2}\pi} \cos^{n-2} x \sin^2 x \, dx$$

$$+ \left(\frac{n}{5}\right) \int_0^{\frac{1}{2}\pi} \cos^{n-4} x \sin^4 x \, dx \dots \pm \left(\frac{n}{2r+1}\right) \int_0^{\frac{1}{2}\pi} \cos^{n-2r} x \sin^{2r} x \, dx + \dots$$

Transforming $\int_0^{\frac{1}{2}\pi} \sin^p z \cos^q z \, dz$ by the substitution $\sin z = \sqrt{x}$, we have

$$dz = \frac{dx}{2\sqrt{x(1-x)}}, \text{ and } \int_0^{\frac{1}{2}\pi} \sin^p z \cos^q z \, dz = \frac{1}{2} \int_0^1 x^{\frac{1}{2}(p-1)} (1-x)^{\frac{1}{2}(q-1)} dx \\ = \frac{1}{2} B\left[\frac{1}{2}(p+1), \frac{1}{2}(q+1)\right].$$

$$\therefore \int_0^{\frac{1}{2}\pi} \sin nx \cot x \, dx, \text{ in beta-functions, } = \frac{n}{2} B\left(\frac{n+1}{2}, \frac{1}{2}\right)$$

$$- \frac{1}{2} \left(\frac{n}{3}\right) B\left(\frac{n-1}{2}, \frac{3}{2}\right) + \frac{1}{2} \left(\frac{n}{5}\right) B\left(\frac{n-3}{2}, \frac{5}{2}\right) - \dots$$

Also solved by C. E. White.

250. Proposed by V. M. SPUNAR, East Pittsburg, Pa.

Differentiate $(\log^n x)$.

Solution by C. E. WHITE, Vanderbilt University, Nashville, Tenn.

$\log^n x = \log \log \log \dots (n \text{ times}) x$.

$$\therefore d \log^n x = \frac{d(\log^{n-1} x)}{\log^n x} = \frac{d \log^{n-2} x}{\log^n x \log^{n-1} x} = \dots = \frac{dx}{\log^n x \log^{n-1} x \log^{n-2} x \dots \log x \cdot x}$$

Also solved by J. Scheffer. Some of our readers misinterpreted the meaning of the notation. It should be remembered that the notation means the log of the log of the log, etc., n times, of x . See Byerly's *Integral Calculus*, 2d Ed., p. 2. ED. F.

251. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Find in terms of x the functions $c_1 x$ and $c_2 x$ defined, respectively, by the relations

$$\begin{aligned} (a) \quad x \log(c_1 x) &= c_1 x \log x, \\ (b) \quad x \log x &= c_2 x \log(c_2 x). \end{aligned}$$

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

(a) We may write (a) thus, $\log(c_1 x)^x = \log x^{c_1 x}$.

$$\therefore (c_1 x)^x = x^{c_1 x}, \text{ or } c_1 x = x^{c_1} = 1 + c_1 \log x$$

$$+ \frac{c_1^2}{2!} (\log x)^2 + \dots + \frac{c_1^n}{n!} (\log x)^n + \dots$$

(b) Similarly, we may write (b), $\log x^x = \log(c_2 x^{c_2 x})$.

$$\therefore x^x = (c_2 x)^{c_2 x}, \text{ or } c_2 x = x^{1/c_2} = 1 + \frac{1}{c_2} \log x + \frac{1}{2! c_2^2} (\log x)^2 + \dots + \frac{1}{n! c_2^n} (\log x)^n + \dots$$

Also solved by J. Scheffer, C. E. White, and V. M. Spunar. Mr. Spuhar, in his solution, used the calculus.

MECHANICS.

131. Proposed by F. P. MATZ.

If the distribution of weight on the foundations of a building is W lb./ (feet)², the foundation must be sunk $D = (W/w) \tan 4(\frac{1}{2}\pi - \frac{1}{2}\psi)$ feet deep in earth of density w lb./ (feet)² and angle of repose ψ .

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let E represent the weight of a portion of a horizontal stratum of earth which is displaced by the foundation of a structure, S the utmost weight of that structure consistent with the power of the earth to resist displacement, ψ the angle of repose of the earth. Then $S/E = [(1 + \sin \psi)/(1 - \sin \psi)]^2$ (see paper "On Stability of Loose Earth," read before the Royal Society on the 19th of June, 1856, and published in the *Philosophical Transactions* for that year). In the problem, $E = Dw$, $S = W$.

$$\therefore \frac{Dw}{W} = \left(\frac{1 - \sin \psi}{1 + \sin \psi} \right)^2 = \left(\frac{1 - \cos(\frac{1}{2}\pi - \psi)}{1 + \cos(\frac{1}{2}\pi - \psi)} \right)^2$$